

## Resources Allocation Strategy in MIMO-SCMA System

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**ABSTRACT:** Sparse code multiple access (SCMA) is one of the most promising non-orthogonal multiple access scheme for future mobile communication. Combined with multiple input, multiple output (MIMO) technique, the spectrum efficiency of SCMA can be further improved. However, due to the non orthogonal feature, existing resource management methods for orthogonal MIMO network can not be applied directly to MIMO-SCMA system. To solve the problem, Four joint codebook and power allocation algorithms are developed in this paper. Simulation results show that the proposed algorithms efficiently solved maximum sum-rate optimization problem and maximum min-rate optimization problem in MIMO-SCMA system.

**Keywords:** MIMO, sparse code multiple access (SCMA), convex optimization, Quality of Service.

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### I. INTRODUCTION

To meet the requirement of massive connectivity, high throughput and low latency in future mobile communication, the non-orthogonal multiple access schemes such as power-domain non-orthogonal multiple access (p-NOMA), sparse code multiple access (SCMA), pattern division multiple access (PDMA), multi-user shared access (MUSA) has been developed [1-3]. The SCMA as a frequency and code domain non orthogonal multiple access scheme that extended from LDS-CDMA [2], can achieve considerable balance between performance and complexity. By taking advantage of the sparse structure of SCMA codebook, the message passing algorithm (MPA) [4] is introduced to separate the non-orthogonal signal. In fact, the spectrum efficiency can be further improved by combining MIMO and non-orthogonal multiple access technique. However, such combination makes it difficult to perform resources allocation. On the other hand, the state of the art resource allocation method for MIMO combined non-orthogonal multiple access mainly focus on power domain MIMO-NOMA, few on code domain MIMO-SCMA. To fill this gap, this paper tries to solve the power allocation problem and code book allocation in MIMO-SCMA system.

In this paper, we formulate the uplink MIMO-SCMA system model with joint factor graph representation [5-6]. Further more, we formulate the sum-rate maximization problem and min-rate maximization problem under this MIMO-SCMA system. To solve the two optimization problem, we decomposed the origin problem into sub-problems and adopt the sequential optimization technique and lower bound approximation technique.

The rest of the paper is organized as follows: Section 2 will discuss the system model for MIMO-SCMA system and formulates the optimization problem for the MIMO-SCMA system. Section 3 will solve the two optimization problem respectively. Simulation results and discussions will be given in Section 4.

### II. SYSTEM MODEL AND PROBLEM FORMULATION

In this section, we briefly review the basic SCMA system model and demonstrate how it can be formulated into the uplink MIMO-SCMA model with joint factor graph representation. The basic SCMA system consists of  $N$  independent data streams and  $K$  orthogonal resources, e.g., OFDM sub-carriers, with  $N > K$  and defines the overload factor by  $\lambda = N/K$  ( $\lambda > 1$ ). Each data stream (user/layer) is assigned with one unique code book  $X_j \in \mathbb{C}^{K \times M}$ , and every  $\log_2(M)$  bits of each data stream are directly mapped into the corresponding codeword from the codebook. A factor graph with  $N$  variable nodes and  $K$  function nodes can be used to represent a SCMA system.

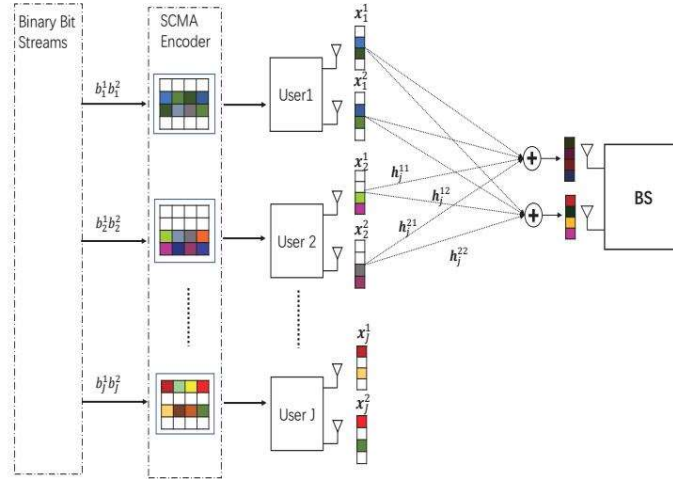


Fig.1: System model of uplink MIMO-SCMA system

We consider a MIMO-SCMA uplink communication system where  $J$  users simultaneously communicate with one base station (BS). Each user is equipped with  $N_U$  antennas and the base station is equipped with  $N_R$  antennas, with system diagram in Fig 1. For the  $n_u$ -th antenna of user  $j$  ( $n_u=1,2,\dots,N_U$ ), every  $\log_2(M)$  bits from the bit stream  $b_j^{n_u}$  are directly mapped into a  $K$ -dimension complex codeword  $\mathbf{x}_j^{n_u}$  from the code book assigned to the  $n_u$ -th antenna, with  $\mathbf{x}_j^{n_u} \in \mathbb{C}^{K \times 1}$ . The entire signal transmitted by user  $j$  can be given as  $\mathbf{x}_j = [(\mathbf{x}_j^1)^T, (\mathbf{x}_j^2)^T, \dots, (\mathbf{x}_j^{N_U})^T]^T \in \mathbb{C}^{N_U K \times 1}$ . The received signal at the base station can be given as

$$\mathbf{y} = \sum_{j=1}^J \mathbf{H}_j \mathbf{x}_j + \mathbf{n}_j \quad (1)$$

$$= \mathbf{H} \bar{\mathbf{x}} + \mathbf{n}. \quad (2)$$

where  $\mathbf{y} = [\mathbf{y}_1^T, \mathbf{y}_2^T, \dots, \mathbf{y}_J^T]^T \in \mathbb{C}^{N_B K \times 1}$ ,  $\mathbf{H} = [\mathbf{H}_1^T, \mathbf{H}_2^T, \dots, \mathbf{H}_J^T]^T \in \mathbb{C}^{N_B K \times N_U K J}$ ,  $\bar{\mathbf{x}} = [\mathbf{x}_1^1, \dots, \mathbf{x}_1^{N_U}, \dots, \mathbf{x}_J^1, \dots, \mathbf{x}_J^{N_U}]^T \in \mathbb{C}^{N_U K J \times 1}$ . And the MIMO channel  $\mathbf{H}_j \in \mathbb{C}^{N_R K \times N_U K}$  is given as

$$\mathbf{H}_j = \begin{bmatrix} \text{diag}(\mathbf{h}_j^{1,1}) & \text{diag}(\mathbf{h}_j^{1,2}) & \dots & \text{diag}(\mathbf{h}_j^{1,N_U}) \\ \text{diag}(\mathbf{h}_j^{2,1}) & \text{diag}(\mathbf{h}_j^{2,2}) & \dots & \text{diag}(\mathbf{h}_j^{2,N_U}) \\ \vdots & \vdots & \ddots & \vdots \\ \text{diag}(\mathbf{h}_j^{N_R,1}) & \text{diag}(\mathbf{h}_j^{N_R,2}) & \dots & \text{diag}(\mathbf{h}_j^{N_R,N_U}) \end{bmatrix}, \quad (3)$$

and the element  $h_j^{nr,nu}$  denotes the channel gain vector for  $K$  OFDM sub-carriers between the  $n_u$ -th antenna of user  $j$  and  $n_r$ -th antennas of BS, and  $\mathbf{n}_j = [(\mathbf{n}_j^1)^T, (\mathbf{n}_j^2)^T, \dots, (\mathbf{n}_j^{N_U})^T]^T \in \mathbb{C}^{N_U K \times 1}$  denotes the additive complex Gaussian noise vector of  $j$ -th user.

By mathematical manipulation, we can reform (1) as

$$\mathbf{y} = \sum_{j=1}^J \sum_{n_u=1}^{N_U} \text{diag}(\mathbf{h}_j^{n_r}) \check{\mathbf{x}}_j^{n_r} + \mathbf{n}_j, \quad (4)$$

Where

$$\mathbf{h}_j^{n_r} = [(\mathbf{h}_j^{n_u,1})^T, (\mathbf{h}_j^{n_u,2})^T, \dots, (\mathbf{h}_j^{n_u,N_U})^T]^T \in \mathbb{C}^{N_R K \times 1},$$

And

$$\check{\mathbf{x}}_j^{n_r} = \underbrace{[(\mathbf{x}_j^{n_u})^T, (\mathbf{x}_j^{n_u})^T, \dots, (\mathbf{x}_j^{n_u})^T]^T}_{N_R} \in \mathbb{C}^{N_R K \times 1}.$$

It can be inferred from (4) that the joint factor graph for MIMO-SCMA system consist of  $N_U J$  variable nodes and  $N_B K$  function nodes. From the perspective of information theory, the sum rate of a communication system is defined as the maximum mutual information between the transmitter and the receiver. The capacity of gaussian channel under power constrain  $P$  is given as:

$$C = \max_{f(x):EX^2 \leq P} I(X;Y) \tag{5}$$

for our MIMO-SCMA system, the capacity can be given as:

$$C_{SCMA} = \max_{\|\mathbf{x}_j^{n_u}\|_2 \leq P} I(\bar{\mathbf{x}}; \mathbf{y}) \tag{6}$$

By taking the conditional entropy, we have:

$$\begin{aligned} I(\mathbf{x}; \mathbf{y}|\mathbf{H}) &= h(\mathbf{y}|\mathbf{H}) - h((\mathbf{y}|\mathbf{H})|\mathbf{x}) \\ &= h(\mathbf{y}|\mathbf{H}) - h(\mathbf{x} + \mathbf{n}|\mathbf{x}) \\ &= h(\mathbf{y}|\mathbf{H}) - h(\mathbf{n}|\mathbf{x}) \\ &= h(\mathbf{y}|\mathbf{H}) - h(\mathbf{n}) \end{aligned} \tag{7}$$

$$\tag{8}$$

According to the entropy maximization theorem [7], the capacity of MIMO-SCMA system is given as:

$$\begin{aligned} C_{SCMA} &= \max_{\|\mathbf{x}_j^{n_u}\|_2 \leq P} h(\mathbf{y}|\mathbf{H}) - h(\mathbf{n}) \\ &\leq \frac{1}{2} \log |2\pi e(\mathbf{H}\mathbf{R}_X\mathbf{H}^H + \sigma_n^2\mathbf{I}_{N_B})| - \frac{1}{2} \log |2\pi e\sigma_n^2\mathbf{I}_{N_B}| \\ &= \frac{1}{2} \log \left| \frac{1}{\sigma_n^2} \mathbf{H}\mathbf{R}_X\mathbf{H}^H + \mathbf{I}_{N_B} \right|. \end{aligned} \tag{9}$$

where  $\mathbf{R}_x$  is the covariance matrix of transmit codewords. By applying equation (4), we have:

$$C_{SCMA} = \log \left| \mathbf{I}_{N_B K} + \sum_{j=1}^J \sum_{n_u=1}^{N_U} \text{diag}(\mathbf{h}_j^{n_u}) \check{\mathbf{x}}_j^{n_u} (\check{\mathbf{x}}_j^{n_u})^H \text{diag}(\mathbf{h}_j^{n_u})^H / \sigma_n^2 \right| \tag{10}$$

$$\geq \log \begin{vmatrix} \bar{\mathbf{R}}_K^1 & \cdots & \cdots & \mathbf{0} \\ \vdots & \bar{\mathbf{R}}_K^2 & & \vdots \\ \vdots & & \ddots & \vdots \\ \mathbf{0} & \cdots & \cdots & \bar{\mathbf{R}}_K^{N_B} \end{vmatrix} = \bar{C}_{SCMA} \tag{11}$$

Where

$$\bar{\mathbf{R}}_K^{n_b} = \begin{bmatrix} R_1^{n_b} & \cdots & \cdots & \mathbf{0} \\ \vdots & R_2^{n_b} & & \vdots \\ \vdots & & \ddots & \vdots \\ \mathbf{0} & \cdots & \cdots & R_K^{n_b} \end{bmatrix}, R_k^{n_b} = 1 + \frac{\sum_{j=1}^J \sum_{n_u=1}^{N_U} |h_{j,k}^{n_b, n_u}| E[(x_{j,k}^{n_u})^2]}{\sigma_n^2} \tag{12}$$

### III. OPTIMIZATION PROBLEMS IN MIMO-SCMA SYSTEM

#### 3.1 sum-rate maximization problem in MIMO-SCMA system

According to previous discussions, the sum-rate maximization problem can be formulated as:

$$\max_{\mathbf{P}, \mathbf{F}} \sum_{k'=1}^{K'} \bar{C}_{k'} = \sum_{k'=1}^{K'} \log \left( 1 + \frac{\sum_{j=1}^J \sum_{n_u=1}^{N_U} |h_{j,k}^{n_b, n_u}| f_{j,k}^{n_u} p_{j,k}^{n_u}}{\sigma_n^2} \right) \tag{13}$$

$$\text{s.t.} \quad \sum_{k=1}^K f_{j,k}^{n_u} = d_f^{n_u} \tag{14}$$

$$\sum_{j=1}^J f_{j,k}^{n_u} = d_v^{n_u} \tag{15}$$

$$\sum_{k=1}^K f_{j,k}^{n_u} p_{j,k}^{n_u} \leq p_{j-max}^{n_u} \tag{16}$$

$$\mathbf{F}_j^{n_u} \in \mathbb{B}^{K, J} \tag{17}$$

$$\forall k' = (n_b - 1)K + k \quad k = 1, 2, \dots, K, \quad n_b = 1, 2, \dots, N_B$$

Where

$$\mathbf{P} = \begin{bmatrix} p_{1,1}^1 & p_{1,1}^2 & \cdots & p_{1,1}^{N_U} \\ p_{2,1}^1 & p_{2,1}^2 & \cdots & p_{2,1}^{N_U} \\ \vdots & \vdots & \ddots & \vdots \\ p_{1,K}^1 & p_{1,K}^2 & \cdots & p_{1,K}^{N_U} \end{bmatrix} \in \mathbf{R}^{+K \times N_U J} \quad (18)$$

The coupled variables  $\mathbf{P}, \mathbf{F}$  and the binary constrain in (17) are the main difficulties to solve this problem. To tackle this, we decompose this problem into two sub-problem respect to each variable, and relax the binary constrain into continuous with regulation term. the corresponding sub-problem are given as:

$$\max_{\mathbf{P}} \sum_{k'=1}^{K'} \log\left(1 + \frac{\sum_{j=1}^J \sum_{n_u=1}^{N_U} |h_{j,k}^{n_b, n_u}| f_{j,k}^{n_u} p_{j,k}^{n_u}}{\sigma_n^2}\right) \quad (19)$$

$$\sum_{k=1}^K f_{j,k}^{n_u} p_{j,k}^{n_u} \leq p_{j,max}^{n_u} \quad (20)$$

$$\forall k' = (n_b - 1)K + k \quad k = 1, 2, \dots, K, \quad n_b = 1, 2, \dots, N_B$$

And

$$\max_{\mathbf{F}} \sum_{k'=1}^{K'} \log\left(1 + \frac{\sum_{j=1}^J \sum_{n_u=1}^{N_U} |h_{j,k}^{n_b, n_u}| f_{j,k}^{n_u} p_{j,k}^{n_u}}{\sigma_n^2}\right) + \text{tr}(\nabla \psi(\mathbf{F}^{(t-1)})^T (\mathbf{F} - \mathbf{F}^{(t-1)})) \quad (21)$$

$$\text{s.t.} \quad (14)(15)(16)$$

$$\forall k' = (n_b - 1)K + k \quad k = 1, 2, \dots, K, \quad n_b = 1, 2, \dots, N_B$$

The objective function for both sub-problem are concave functions, and all constrain in the sub-problems are linear constrains, thus both of the sub-problems are convex optimization problem. As the result, the maximum sum-rate optimization problem can be solved by sequentially solves the two sub-problems with convex optimization programming software package like CVX[8].

### 3.2 Min-rate maximization problem in MIMO-SCMA system

In the previous section, we solved the maximum sum-rate optimization problem by sequentially solves sub-problems. However, the proposed maximum sum-rate algorithm tends to allocate more power to users with better channel condition, few to users with poor channel condition. In order to achieve a relatively fair allocation, we propose a maximum min-rate algorithm for MIMO-SCMA system.

According to previous section, the maximum min-rate optimization problem in MIMO-SCMA system can be formulate as:

$$\max_{\mathbf{P}, \mathbf{F}} \min_j \bar{C}_j = \sum_{k'=1}^{K'} \log\left(1 + \frac{\sum_{n_u=1}^{N_U} |h_{j,k}^{n_b, n_u}| f_{j,k}^{n_u} p_{j,k}^{n_u}}{\sigma_n^2 + \sum_{l=1}^{j-1} \sum_{n_u=1}^{N_U} |h_{l,k}^{n_b, n_u}| f_{l,k}^{n_u} p_{l,k}^{n_u}}\right) \quad (22)$$

$$\text{s.t.} \quad \sum_{k=1}^K f_{j,k}^{n_u} = d_f^{n_u} \quad (23)$$

$$\sum_{j=1}^J f_{j,k}^{n_u} = d_v^{n_u} \quad (24)$$

$$\sum_{k=1}^K f_{j,k}^{n_u} p_{j,k}^{n_u} \leq p_{j,max}^{n_u} \quad (25)$$

$$\mathbf{F}_j^{n_u} \in \mathbb{B}^{K, J} \quad (26)$$

$$\forall k' = (n_b - 1)K + k \quad k = 1, 2, \dots, K, \quad n_b = 1, 2, \dots, N_B$$

$$\forall j = 1, 2, \dots, J$$

unfortunately, unlike the sum-rate maximization problem, the objective function in this optimization problem itself is a non-convex non-concave function, thus both of its sub-problems are non-convex even with continuous relaxation. To tackle this problem, we can consider the lower bound for the objective function by utilizing the property for logarithm function. Further more, an auxiliary variable  $t$  is introduced to solve this problem. After the lower bound is taken, the corresponding sub-problems are given as:

$$\begin{aligned}
 \max_{\mathbf{P}, t} \quad & t \\
 \text{s.t.} \quad & t \leq \sum_{k'=1}^{K'} \log(\sigma_n^2 + \sum_{l=1}^j \sum_{n_u=1}^{N_U} |h_{l,k}^{n_b, n_u}|^2 f_{l,k}^{n_u} p_{l,k}^{n_u}) \\
 & -\varphi_P(\mathbf{P}^{(t-1)}) - tr|\nabla\varphi_P(\mathbf{P}^{(t-1)})(\mathbf{P} - \mathbf{P}^{(t-1)})| \quad (27)
 \end{aligned}$$

$$\begin{aligned}
 & \sum_{k=1}^K f_{j,k}^{n_u} p_{j,k}^{n_u} \leq p_{j,max}^{n_u} \quad (28) \\
 & \forall j = 1, 2, \dots, J
 \end{aligned}$$

And

$$\max_{\mathbf{F}, t} \quad t + \psi(\mathbf{F}^{(t-1)}) + tr|\nabla\psi(\mathbf{F}^{(t-1)})(\mathbf{F} - \mathbf{F}^{(t-1)})| \quad (29)$$

$$\begin{aligned}
 \text{s.t.} \quad & t \leq \sum_{k'=1}^{K'} \log(\sigma_n^2 + \sum_{l=1}^j \sum_{n_u=1}^{N_U} |h_{l,k}^{n_b, n_u}|^2 f_{l,k}^{n_u} p_{l,k}^{n_u}) \\
 & -\varphi_F(\mathbf{F}^{(t-1)}) - tr|\nabla\varphi_F(\mathbf{F}^{(t-1)})(\mathbf{F} - \mathbf{F}^{(t-1)})| \\
 & 0 \leq f_{j,k}^{n_u} \leq 1 \quad (30)
 \end{aligned}$$

$$\forall k' = (n_b - 1)K + k \quad k = 1, 2, \dots, K, \quad n_b = 1, 2, \dots, N_B$$

By using lower bound approximation technique, now both of the objective function in the sub-problem become a concave function, and all constrain in the sub-problems are linear constrains, thus both of the sub-problems are convex optimization problem. Therefore, the min-rate maximization problem can be solved similar to previous section.

### 3.3 Heuristic Algorithm for resource allocation in MIMO-SCMA system

In the previous section, we proposed two algorithms: sum-rate maximization algorithm and min-rate maximization algorithm, which solve the corresponding problem by sequentially solves convex optimization sub-problem. However, the complexity for solving convex optimization sub-problem in these proposed algorithms is relatively high, and its complexity grows rapidly when the system scale becomes larger. To strike a balance between performance and computational complexity, in this section we propose two low complexity heuristic algorithm for both sum-rate and min-rate maximization problem.

The idea for the heuristic algorithm is simple, we consider a single step optimization problem, each step we add one element into the valid position in the joint factor graph matrix to maximize the objective function. For the sum-rate maximization problem, we search all valid elements in the joint factor graph matrix where element 0 can be turn into 1, and choose one that maximize the sum-rate. Similarly, in the min-rate maximization problem, we choose one valid element in the joint factor graph matrix to turn it from 0 to 1 which maximize the min-rate.

Obviously for the heuristic algorithm above, the complexity only depends on the size of the joint factor graph matrix, which means that its complexity grow linearly with the number of users and the number of antennas at both transmitter and receiver side.

## IV. SIMULATION RESULTS

In this section, we evaluate the proposed algorithm in the MIMO-SCMA system, under the configuration that is  $J=6, K=4, M=4, N_U = 2, N_R = 2$ . Without loss of generality, we assume that each antenna equipped by the same user is assigned with the same codebook, and the results are averaged from  $10^3$  times of independent realization of rayleigh fading channel.

For the sake of Simplicity, we call the algorithms in Fig.2 and Fig.3 from top to bottom as algorithm 1 - algorithm 4. As we can see in Fig.2, all four algorithms have ideal convergence behavior, algorithm 1, 3, 4 have relatively high convergence rate and converges to 95 percent of its top performance with 5 iterations. we define the maximum achievable sum-rate of algorithm 1 as the baseline with ratio 1, thus the ratios of algorithm 2, 3, 4 are 0.9, 0.85, 0.7 respectively.

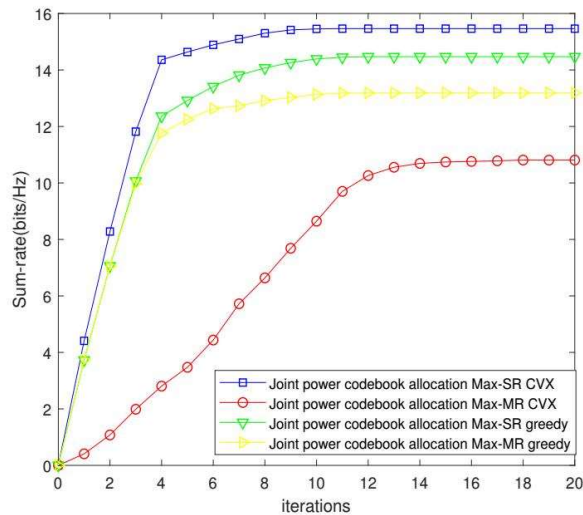


Fig.2:Sum-rate vs iterations

Next, in Fig.3 we analysed the relation between min-rate and iterations. As we can see, algorithm 2 can achieve the highest min-rate, similarly we define the maximum achievable sum-rate of algorithm 1 as the baseline with ratio 1, thus the ratios of algorithm 1, 3, 4 are 0.6, 0.35, 0.3 respectively.

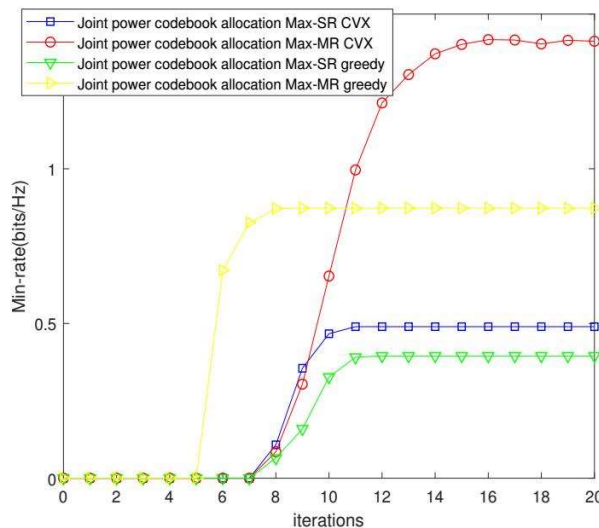


Fig.3:Min-rate vs iterations

## V. CONCLUSION

This paper formulates the sum-rate maximization problem and min-rate maximization problem for MIMO-SCMA system. And solved the two optimization problems by sequential convex programming as well as their low complexity heuristic algorithm. Simulation result showed that the proposed convex optimization based algorithm achieves good performance with relatively higher complexity, while the greedy algorithm suffer a bit performance loss, but has much lower computational complexity.

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